ESSENTIAL SERVICES COMMISSION

TAXI FARE REVIEW 2013-14

Staff paper – Developing an industry model

AUGUST 2014

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PREFACE

On 31 March 2014, the Essential Services Commission completed its 2013-14 taxi fare review and provided a final report to the Minister for Public Transport. As part of this review, we surveyed operators and drivers to better understand industry costs and driver behaviour and incentives. We were also able to use detailed trip data that network service providers submitted to the Taxi Services Commission.

Overall the information from the above sources provided a reasonable picture of the Metropolitan taxi industry and formed the basis for the Commission to estimate the required average fare increase.

As part of the 2013-14 review, the Commission also embarked on developing an industry model that attempts to estimate market outcomes, that is, the supply of, and demand for, taxi services under different fare scenarios. Given the complexity of the various interrelationships within the taxi industry, it is impossible to model all variables and adequately predict the results of price changes. As a result, the industry model was only one input into our fare analysis. This paper summarises the high level workings of the industry model.

As part of our future consultation processes, we intend to seek feedback from interested stakeholders on how best taxi prices can be modelled to assist us in our next taxi fare review.

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1 CONTEXT FOR PRICE SETTING

1.1 BACKGROUND AND PURPOSE

The Essential Services Commission is Victoria's independent economic regulator of certain prescribed services, as determined by Government.

In February 2014 we received terms of reference from the Minister for Public Transport to complete a review of taxis fares in Victoria. The review covered the Metropolitan, Outer Suburban, Urban and Country taxi zones[1].

We completed this review and provided the Minister for Public Transport with our report on 31 March 2014.[2] We proposed an average fare increase of 12.5 per cent. On 11 April 2014 the Victorian Government announced that our proposed fares would apply from 19 May 2014.

This high level paper regarding the development of an industry model was foreshadowed in chapter 6 of our final report and is now released for further discussion with the industry.

1.2 DESIGNING TAXI FARES

The taxi market is complex and dynamic, and the relationship between prices and revenue is not straightforward. In order to assist the Commission's analysis regarding taxi fares, the Commission released a paper[3] that set out twelve principles to guide us in our decision. These principles underlie the need to balance the long term interests of

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[1] Zone boundaries changed on 1 July 2014.

[2] Essential Services Commission 2014, *Taxi fare review 2013-14 – Final Report*, March.

[3] Essential Services Commission 2013, *Taxi fare review 2013-14 – Principles Paper*, October.

Victorian consumers with the financial viability of the industry. That is, fares must be set high enough to cover operator's costs and to make it worthwhile for drivers to take on shifts, but not so high that passengers are discouraged too much from using taxis.

Fare setting in this environment involves a careful consideration of the incentives for the different participants. A dynamic model allows us to consider these interactions in a consistent and systematic way by expressing the relationships between price, supply and demand mathematically.

1.3 PRICE SETTING PROCESS

The Commission used a number of tools to inform its taxi pricing, some quantitative such as models, and some qualitative, such as stakeholder consultation. As stated in our final report, we approached our price setting role by looking first at the fare *level*, and secondly at the fare *structure*.

Setting the fare level involved finding the overall change in prices required to maintain industry viability – in this case a 12.5 per cent increase. To come to this average increase we studied the costs facing taxi operators. The Commission undertook a survey, used advice from consultants and engaged in significant consultation with operators to arrive at this figure.

Setting the fare structure involved applying that 12.5 per cent average increase across fare components and times of day. It is a complicated task, as there are infinite options available to deliver an average fare increase, and each choice requires some trade-offs between the relative impacts on different participants in the taxi industry.

We used a model of prices and demand to test many ways to distribute the 12.5 per cent across fare components and time periods using the latest 2012 trip data. This is a basic model which applies different prices to existing patterns of trips to show the impact on total revenue. This model is simple, and allows many scenarios to be created, amended and re-tested very quickly. It does not account for the effect of changes in prices on the supply of taxis available or alterations to the pattern of demand.

We also developed a dynamic industry model to consider market outcomes from different fare structures and levels.[4] We used the dynamic model to look at the implications of different price structures on the different participants – customers, drivers and operators. The results of the model help us to understand the supply and demand side implications of different prices.

1.4 DATA FOR THE MODEL

To build the model of industry supply and demand we utilised available information on taxi trips and driver shifts, and collected information on the costs of operating a taxi. We had access to data on trips which was provided by Network Service Providers (NSPs). The data included time, distance and location details for all booked, rank and street hail taxi trips between 2011 and mid-2013. It should be noted that the data did not include the fares for these trips, and the Commission relied on a standard calculation[5] to estimate revenue from fares.

As a separate data set, the Commission also received details of driver shift start and end times for 2011 and 2012. This allowed us to observe trends in driver shifts and to calculate the number of taxis in service at any given time.

To understand what factors influence the supply of taxis, significant work was undertaken to assess the costs facing the taxi industry. A survey focussing on the costs of providing taxi services was sent to all operators in the metropolitan, outer suburban and urban taxi zones (over 2000 operators). We received 275 responses, covering over 1150 taxis. Following analysis of the survey information and estimation of taxi cost profiles (e.g. for standard and wheelchair accessible taxis in the metropolitan zone) we met with operators to test and refine the cost profiles. (Chapter 3 of our *Taxi fare review 2013-14 - Final Report* provides detailed discussion of the estimation of the taxi cost profile.)

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[4] We engaged the Centre for International Economics (CIE) to develop a model of the metropolitan taxi industry. The model uses mathematical equations to simulate decisions of operators, drivers and customers under different fare scenarios.

[5] The calculation assumes 24 seconds stationary for every kilometre travelled and 2 minutes waiting time for each journey. It is based on the TSC's fare estimator.

2 HOW THE INDUSTRY MODEL WORKS

The industry model has three sets of equations which solve sequentially to represent the supply and demand effects of different prices. Appendix A contains the technical explanation of the equations. In this chapter we will summarise the key aspects of this model.

2.1 TIME STRUCTURE

The model's structure was based on patterns of demand shown in the trip data. The hourly demand profile by day of the week is fairly consistent throughout the year, but differs noticeably in January and December. To take into account these patterns, the model uses three different profiles to represent January, December, and February to November. Results are calculated for a representative week for each period and extrapolated out to create total demand for a calendar year.

2.2 DEMAND

The demand equation calculates the total kilometres of taxi travel that will be demanded for the price structures entered. The model is calibrated to a base demand profile from existing trip data which is adjusted to the new conditions implied by the prices. The demand equation is run for each hour of the week across the three time periods and is dependent on two key assumptions; elasticity and the value of time.

ELASTICITY

Elasticity of demand is a measure of how consumers will respond to changes in prices and waiting time. For most goods and services, an increase in price will result in a reduction in demand as consumers will forgo the product or find substitutes. The taxi model allows us to input an elasticity assumption which can vary according to the time of day and day of the week. Historically it has been problematic to settle on an elasticity assumption for taxi use, as consumers respond to price differently depending on the nature of the particular journey, and all assumptions must combine different users' elasticity into an average.

As part of the Taxi Industry Inquiry (TII) in 2012 the Hensher Group completed a study of the demand for taxi and hire cars in Melbourne.[6] The study used a sample across five categories of use (business traveller, tourism, general day-to-day activity, night time travel, and Multi Purpose Taxi Program (MPTP)) to examine behavioural influence on traveller choice for specific trips in the Melbourne metropolitan region.

The price elasticity calculated in the Hensher study ranged from -0.605 to -1.478 with a weighted average of -1.042, but the report noted that due to the complex non-linear equations used in the study, the averages are "not meaningful", and qualified its elasticity results as "illustrative, but substantive". This is despite the fact that the Hensher study was one of the most elaborate studies of this market to date, and indicates the difficulty of calculating price elasticity. Booz Allen Hamilton, in a report for IPART in 2003, cited a range of elasticity between -0.2 and -1.0 from an assessment of international studies.

Estimating the consumer response to changes in taxi fares is very complicated, and the results of studies vary widely. The model allows elasticity to be entered as a variable, and for different values to be applied to different times of day.

VALUE OF TIME

The value of time (VoT) represents the opportunity cost of passenger waiting time. In the demand equation it adjusts consumer demand for taxi trips in response to changes

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[6] The Hensher Group 2012, *Demand for Taxi and Hire Car Services in Melbourne, Victoria*, April.

in waiting time. The demand equation operates such that reduced waiting time stimulates new demand. The magnitude of that demand response depends on the assumptions for the value of time and elasticity of demand. By default, the model uses a VoT of $57 per hour, which is the value implied by the Hensher report, but it can be set at any value.

2.3 ENTRY (SUPPLY)

The entry equations calculate the number of taxis that can make reasonable returns under a particular set of prices. The equations assume that this is the number of taxis that will choose to be available in that time period. Supply is calculated at three interdependent levels; hourly, by shift, and annually. In reality, operators are not confined to any set structure for driver shifts, but the most common pattern is for a car to operate one or two shifts in each 24 hour period. The changeover is most commonly around 3 pm. The model uses two shifts of 12 hours changing over at 3pm for the entry equation. Taxis may only work for part of a shift.

The hourly supply cannot exceed the number of taxis that have entered a shift. A shift is limited to the total number of licensed taxis. The maximum number of taxis cannot be higher than the number of licences[7], but if prices are sufficient to generate excessive returns[8], the number of licences will rise assuming that new entry will be available under the "as of right" permit reforms which commenced in July 2014.

2.4 QUALITY (WAITING TIME)

Waiting time represents an average figure for passengers hailing a taxi in the street. It is calculated in the model based on taxis available and trips demanded in a particular hour or shift. Waiting time is lowest in periods where there is low demand and a large number of taxis on the road. The waiting time equation adjusts average waiting time

[7] The opening number of taxis is set at 4370. This is the January 2014 figure of 4 310, plus the 60 new permits issued in early 2014. At the time of modelling it was not known that only 30 of these permits would be issued.

[8] The underlying cost structure assumes a return to operators of 14.5 per cent as per our pricing decision

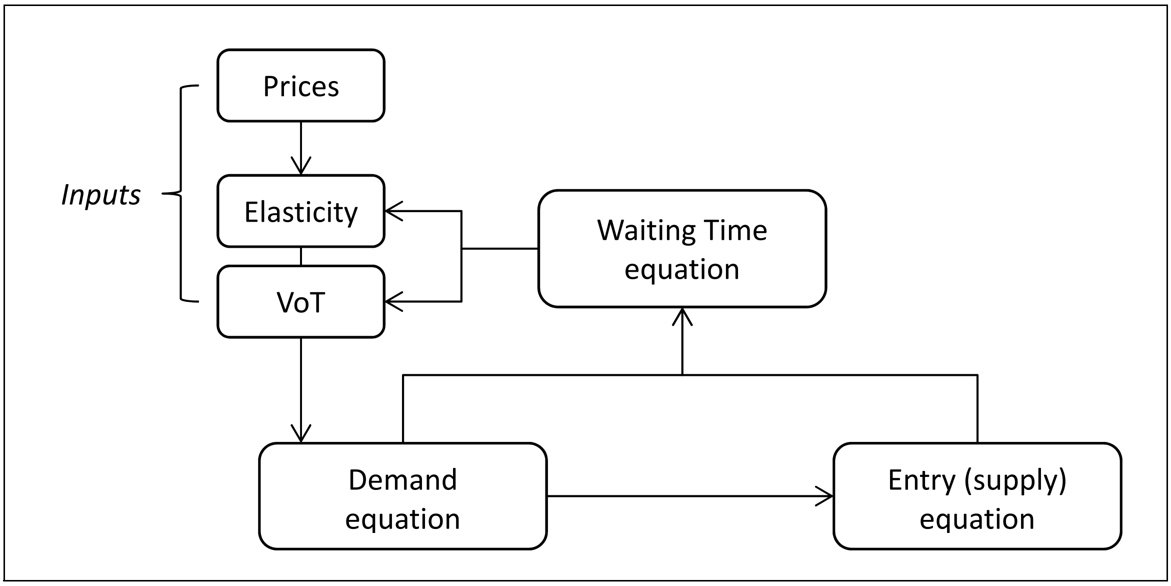
based on the underlying trip and shift data to account for the changes to supply and demand implied by new prices. Changes in the waiting time feed back into the demand equation.

2.5 SOLVING THE MODEL

The model is designed to simulate the impacts of the supply of and demand for taxis under different price scenarios. In order to do this, iterative changes to waiting time, demand and supply are fed back into the same equations that calculate waiting time, demand and supply. For example, waiting time is a product of the supply and demand equations, but is also an input to the demand equation. Figure 2.1 illustrates the circularity of the equations.

The model solves to a point where the constraints of the demand, entry and waiting time equations are all met under a set of inputs (prices, elasticity and VoT). A total of 1 555 equations are run to reach that equilibrium point.

FIGURE 2.1 MODEL STRUCTURE
(Simplified for illustration purposes)



3 UNDERSTANDING THE MODEL'S OUTPUTS

3.1 MODEL OUTPUTS

A range of outputs are calculated by the model (table 3.1). These outputs can be used to assess price scenarios.

TABLE 3.1 DESCRIPTION OF OUTPUTS

Industry revenue	The total revenue from fares for a year. This includes revenue from booking fees, flagfall, distance and waiting charges. Revenue is calculated each hour based on the modelled demand.
Taxi revenue	Average revenue per taxi. This is the industry revenue divided by the number of licensed taxis. The number of licensed taxis can change based on the entry equation.
Occupancy rate	Average overall utilisation – total minutes of trips / total minutes of taxis on shift. This is calculated hourly in the model, and presented as an annual average for assessment of scenarios.
Passenger kilometres	Based on the modelled trip data, this sums the total number of metered passenger kilometres travelled.
Average waiting time	The industry model estimates the average passenger waiting time for street hail (the "cruising market") . Waiting time is calculated hourly based on the modelled demand and supply of taxis available. Waiting time outcomes illustrate the price/service trade-offs that exist under different scenarios.
Consumer welfare	Measures how much consumers value the taxi services taken, over and above the level of the fare paid. It quantifies the consumer outcome of a scenario in dollar terms. The VoT is a key input into this measure.
Total welfare	A measure that combines consumer welfare with producer welfare (producer welfare measures the value of fare revenue over and above the cost of providing taxi services). It quantifies the outcome of a scenario for all participants, in dollar terms.

3.2 THE RELATIONSHIP BETWEEN FARE INCREASES AND REVENUE

To illustrate the relationship between revenue and prices we modelled six scenarios based on incremental increases to the fare structure as it stood prior to our review.[9] The first two outputs measure the effect of price on industry revenue and average taxi revenue, which rise as prices go higher. The next two measures – occupancy and passenger kms – decrease as prices rise. This is the effect of elasticity which lessens customer demand for taxis as the cost of travel gets higher.

TABLE 3.1 TESTING DIFFERENT FARE LEVEL INCREASES

	0%	5%	10%	15%	20%	30%
Taxi revenue ($/taxi)	172 169	177 412	180 102	183 421	182 711	177 491
Industry revenue ($m)	746	769	787	802	817	838
Occupancy rate (%)	32.3	31.0	29.8	28.2	27.5	25.1
Passenger kms (mill)	306	300	294	287	280	267
Waiting time (mins)	11.9	11.4	11.0	10.6	10.2	9.6
Consumer welfare ($m)	490	471	455	434	414	375
Total welfare ($m)	609	605	599	582	575	536

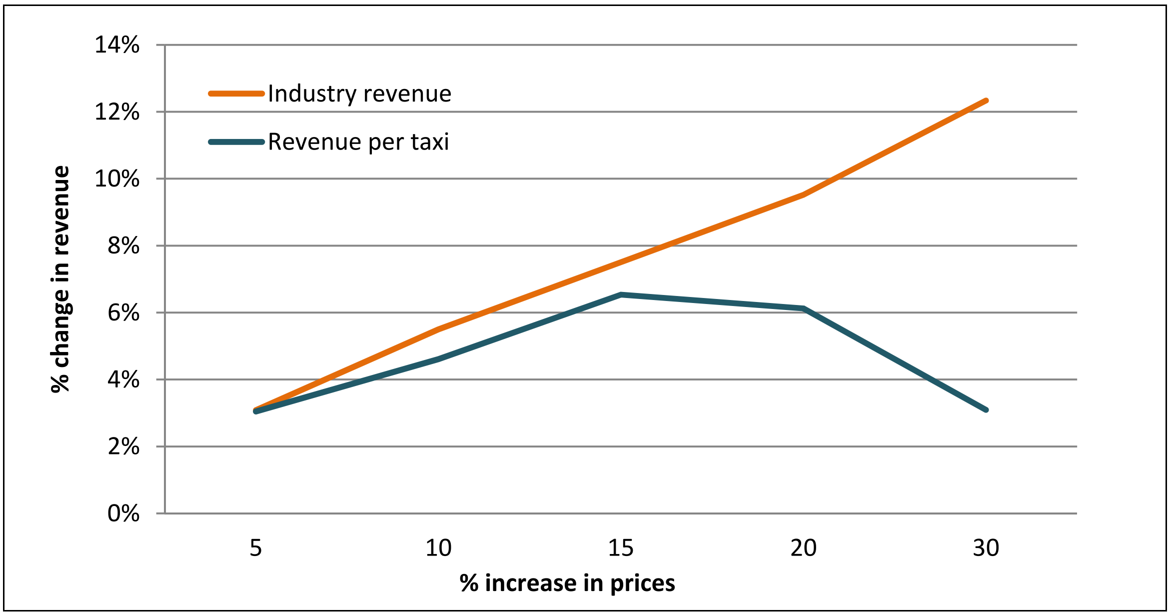
Passenger waiting time is reduced as prices rise. This is because fewer customers demand taxis and at the same time more taxis are on the road. With higher prices, taxis will still make reasonable returns, even if some demand is lost. The taxi revenue results show that with price increases greater than 15% new taxis enter the market. This is best illustrated in Figure 3.1, which shows the relative increase in revenue to the industry and to the average taxi as prices are raised.

The results in table 3.1 also show that with higher fares, occupancy rates and passenger kilometres decrease — more taxis on the road per shift are taking fewer trips. Passenger waiting times also decrease for the same reasons — there are more taxis on the road and demand for taxis reduces as fares increase. Finally, consumer welfare decreases as fares are increased (since fewer trips are taken, and for those

[9] Elasticity is set at –0.8 and the Value of Time is $57 per hour.

trips taken, a higher fare is paid), as does total welfare (increases in producer welfare from higher prices are offset by reductions in consumer welfare).

FIGURE 3.1 REVENUE VS. FARE INCREASE



3.2.1 SENSITIVITY ANALYSIS

As with any model, the results are dependent on the information and assumptions that underpin the calculations. In the examples above the assumptions for elasticity and VoT are the same for each scenario. To test the sensitivity of the model to these assumptions we ran a series of 20 simulations where the price structure was held constant[10], but five different values for elasticity were tested against four different VoT figures.

[10] Sensitivity was tested using a 10% price increase based on the metropolitan fares in place prior to our decision

FIGURE 3.2 SENSITIVITY OF DEMAND TO ELASTICITY AND VoT

Based on 10% increase in prices

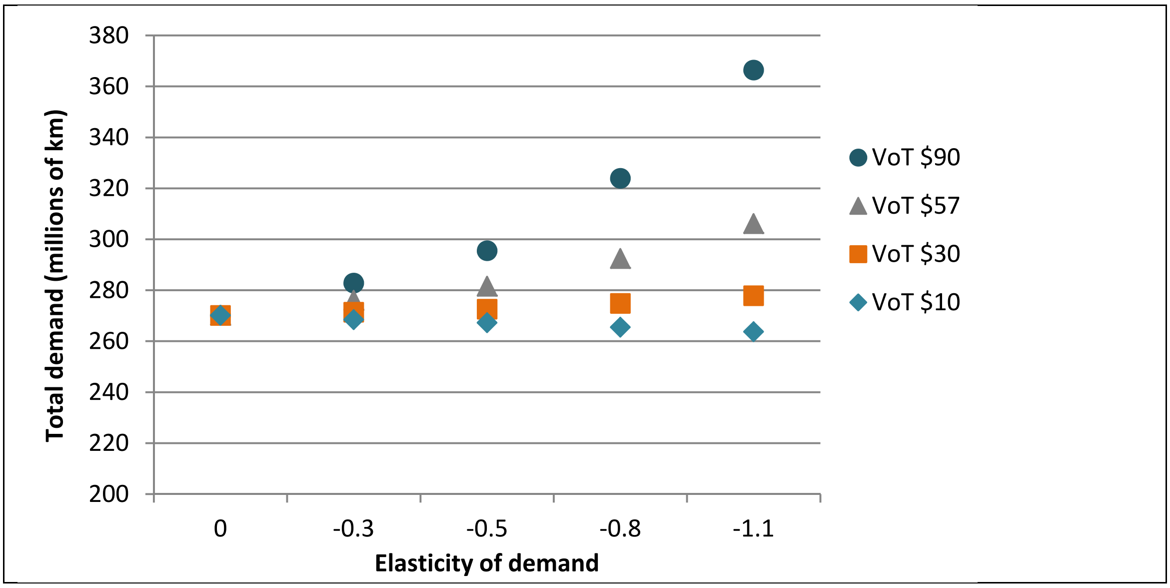


Figure 3.2 illustrates the influence of these two assumptions on the modelled demand for taxi trips (shown here in total km). Where VoT is set at $10, overall demand goes down as the market is made more elastic. When VoT is set at $57 and $90, demand rises as the market is made more elastic. The additional demand in the latter cases highlights the response to waiting times in the demand equation. The higher customers are assumed to value their time, the more they will respond to reductions in waiting times by taking more taxi trips.

The model is very sensitive to elasticity and value of time, and it is crucial to have reliable assumptions for these. The Hensher report published an average elasticity of -1.1 and an implied a VoT of $57. The IPART taxi model in 2013 used elasticity of -0.8 and a VoT of $30. In our sensitivity analysis demand is eleven per cent higher under the Hensher scenario compared to IPART's assumptions.

3.2.2 INTERPRETING THE RESULTS

The Commission used the model to help understand the trade-offs between consumer and operator outcomes, but finding the right balance still rests very strongly on input from stakeholders and adherence to the principles outlined in our Principles Paper.

Analysis showed us that the results were extremely sensitive to assumptions of elasticity and VoT and in both cases coming up with an acceptable value for these inputs was difficult (particularly in the time available in our review).

It is also clear that elasticity of demand and VoT vary greatly depending on the individual circumstances of the passenger. A business traveller during the day is in quite a different situation from a person taking a taxi home from the city late on a Saturday night. The VoT can only be entered as a single input, and generalising for this value is problematic given its influence on results.

The results reported in section 3.2 show that the model's results make sense when comparing different fare levels. But given the sensitivity to elasticity and VoT, the Commission placed less emphasis on the model when comparing different fare structures.

4 FUTURE DIRECTION

As the previous discussion shows, there are a number of limitations in using the results from a model to evaluate price options without undertaking considered qualitative analysis of the price structures. This is the case when using any model when trying to replicate the real world. The Commission therefore is mindful of these issues when assessing the results from the model.

For the 2014 review of prices we identified a number of factors that suggested we take a cautious approach when evaluating the results from the model.

- The model does not represent submarkets for fares of different distances. Other evidence suggests drivers have a preference for some trips over others.
- The model overstates driver take up of late night shifts in peak times. We know strong incentives are required to improve supply at these times, while the model returns the maximum supply of taxis on Friday and Saturday nights even in scenarios with relatively low peak fares.
- Information on actual waiting time and the variation of waiting time under different supply and demand conditions is not currently available to validate this aspect of the model.
- Results are more sensitive to the overall change in prices than to the structure of fares, and the range of results for different fare structures can be marginal.
- The model relies on a universal measure for VoT when this varies greatly depending on location, customer type and other circumstances.

Because of these limitations the Commission was reluctant to make decisions relying solely on the outcomes of the model.

In the future, the Commission may undertake a study of the demand patterns in metropolitan taxi use. This will cover temporal and geographic distinctions in an effort to find more targeted equations that represent the full range of taxi use. The next tranche of trip data from NSPs will give a first indication of the supply and demand response to the new fares introduced in May 2014. This data should be particularly useful in further estimating a value for consumer elasticity and will provide a strong basis for segmenting and weighting other consumer inputs for our future modelling.

4.1 CONCLUSIONS

The taxi industry is complex, and even with access to very detailed data on taxi trips and driver behaviour it is difficult to adequately predict the results of price changes with great confidence. The taxi market is made up of many trips of different characteristics and circumstances — length, destination, time of day, weather and myriad other factors influence the decisions of customers and drivers in making a taxi journey. A model will always be limited in its ability to predict the outcome of price changes as it necessarily simplifies complex consumer behaviour and producer decisions. While the Commission is committed to the use of dynamic modelling in future pricing decisions to inform itself of impacts from different fare scenarios, it will be careful in the use of any modelling results as a model cannot capture all aspects of the industry, and so results need to be interpreted with care.

In preparation for our next review we are considering the best means to model the industry to meet our objectives. In that review, we will have further driver and customer data and will be able to assess the response to the recent price change. We intend to incorporate the new data into our future modelling and will be giving considerable thought to the ways that we can better use economic and statistical techniques to inform our next pricing decision. Going forward we have the option to focus our modelling on particular aspects of the market that require attention — such as supply at peak times and the market for short fares — as well as providing an overall industry picture. We will be seeking feedback from stakeholders on the best approach.

5 APPENDIX A — BEHAVIOURAL EQUATIONS

Behavioural equations

A model of the taxi industry should involve the following components.

- A **demand equation** that links the quantity of services demanded to the price of taxi services and the quality of taxi services.
- An **entry equation** that determines the number of taxis choosing to operate for a particular hour and shift and the number of taxis choosing to operate at all.
- A **quality equation** that links the quality of services to the number of taxis available and not utilised. Quality of services most directly related to taxi numbers is the waiting time. Potentially, other quality measures could be also change with changes in taxi availability.

The set-up of a taxi industry model differs from 'standard' industry models because the price is regulated, the costs are influenced by Government and because of the feed-back of quality (i.e. waiting time) to demand.

1. Demand equation

The demand for taxi services could be measured as the number of trips, the number of trip kilometres or some combination of these. We propose to measure demand as the number of trip kilometres, following the approach in Rouwendal et al 1998[1] and used in the Victorian Taxi Inquiry and IPART taxi model.

Demand reflects the price of taxi services and the waiting time. For example, a linear demand specification would be.

$$Q^t = {C_Q}^t - {\beta_P}^t.P^t - {\beta_W}^t.W^t$$

Where Q is the quantity of taxis demanded (in trip kilometres), P is price per kilometre and W is waiting time for the taxi user (in minutes). C_Q is a constant calibrated to match the observed demand for each model period, β_P is the responsiveness of demand to price and β_W is the responsiveness of demand to waiting time. The time period is t.

We set the price as a price per passenger kilometre. This is a combination of the flagfall, kilometre price and waiting price determined by the ESC.

The responsiveness of demand parameters (β_P and β_W) are calibrated to match estimates of elasticities of demand and the value of time from the Hensher Group survey for the Victorian Taxi Inquiry for these parameters.

- The Victorian Taxi Inquiry's draft report suggested an elasticity of around -1 for Melbourne based on the Hensher Group study.
- Booz Allen Hamilton, in a report for IPART in 2003, noted that the majority of international studies reported a demand elasticity of -0.2 to -1.0.[2]

The response to waiting time should reflect the response to fares, as time and money are to some degree substitutes. The Hensher Group study implies a value of time of around $57 per hour.

The demand equation identified above is aggregated across different trip locations and distances — it is a composite demand. We have considered breaking up demand into different trips. However, the information on the responsiveness of demand is not sufficiently robust to apply different responsiveness to these different trip segments.

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1 Rouwendal, J., H. Menk and P. Jorritsma (2008), *Deregulation of the Dutch taxi sector,* European Transport Conference.

2 Booz Allen Hamilton 2003, *Appraisal of taxi fare structure issues,* p. 10.

The demand equation is for a specific hour of the week (and month).

The model allows for different elasticities across different time periods. For the base specification, we apply an elasticity of demand estimate of 0.8 for all time periods and test the impact of varying this assumption. The model does allow for the entry of different demand elasticities for different time periods, which are also tested.

We use a base estimate of the value of time of $57 per hour. (This could be for multiple passengers. For example, if two people were waiting then this would imply a value of time of $28.5 per person.) We also test the influence of this assumption.

2. Entry equations

There are a number of entry equations relevant for the Victorian taxi industry.

The first is entry for a particular hour t. The number of taxis on the road for a given time period should reflect the revenues available from operating in that time period and the costs.

For a driver, the profit from a time period (π_D^t) is:

$$\pi_D^t = s.P^t.q^t - c_D.q^t - F_D$$

Where s is the share of revenue to the driver, P is the price per passenger kilometre, q is the quantity of passenger kilometres per taxi, c_D is the cost of a passenger kilometre for the driver, $F_D$is the fixed cost of making the taxi available in a given hour and t is the time period. In reality it is likely that drivers face no marginal costs for additional passenger kilometres ($c_D = 0$) and this is assumed in the model.

For an operator, the profit (π_O^t) from a time period is:

$$\pi_O^t = (1 - s).P^t.q^t - c_O.q^t - F_O$$

Where c_O is the operator cost per passenger kilometre and F_O is the operator cost of making the taxi available in a given hour.

The actual costs for drivers and operators means that:

$$\pi_O^t > \pi_D^t$$

This means that when a taxi is bailed from an operator to a driver, it is the driver who decides the pattern of hours when a taxi is available.

Not all taxis are bailed – some are based on an operator-driver model. In this case, the profit for an operator driver is:

$$\pi_D^t + \pi_O^t = P^t.q^t - c_D.q^t - F_D - c_O.q^t - F_O$$

We do not use this as the basis for decisions as this model does not calibrate well to actual entry and we would anticipate that the bailee would be the marginal entrant. This is simply because not all taxis can be driven under an operator-driver model because there are more taxis than operators.

We allow for a mildly upward sloping supply curve for drivers but not operators. This implies that higher earnings are required to bring more taxi drivers into working an hour.

$$F_D = F_{D,0} + F_{D,1}.T^t$$

Where $F_{D,0}$ is a fixed cost component and $F_{D,1}$ is a constant relating the number of taxis to costs. T^t is the number of taxis operating for that time period.

Note that F_0 and F_1 are allowed to differ by time period. We allow some time periods to be standard and some to require a premium for earnings. Our analysis suggests that a premium is required for early morning hours (3am to 5am) and probably for weekends.

The number of taxis available for an hour must be less than the number that make themselves available for a shift. The shift decision is again, because of the structure of costs, driven by bailee profits. The shift structure allows for a bailee to require some level of earnings to decide to rent the taxi for a particular shift, in addition to the amount required for each hour they keep the taxi on the road. This is evidenced by drivers tending to bail a taxi for periods of over 6 hours, rather than just bailing a taxi for the peak hours.

The driver profit for a shift (${\pi_D}^S$) is

$$ {\pi_D}^S = \sum_{All\ hours\ in\ shift\ t} {\pi_D}^t . \frac{T^t}{T^S} - C^S $$

Where T^S is the number of taxis entering the shift, T^t is the number entering each hour of the shift and C^S is a driver cost per shift.

Alongside the driver entry equations is also an entry equation for a taxi to obtain a licence at all, based on aggregate profit for the operator.

$$ {\pi_O}^y = \sum_{All\ shifts\ t} {\pi_O}^t . \frac{T^t}{N} - C^y $$

Where N is the total number of taxi licences. Note that C^y includes a lease cost.

Entry will occur in a time period under a number of alternative possible constraints:

- Let T* be the number of taxis where $\pi_D^t = 0$
- Let T^ be the number of taxis where ${\pi_D}^S = 0$
- Let N be the number of taxis where ${\pi_O}^y = 0$

ThenT^t is equal to the minimum of T*, T^ and λN. Note that the constraint is λN where λ~0.9 because some taxis are off the road at any point due to repairs and maintenance.

To solve this in practice then we solve for each of the three conditions as follows.

For each hour, we allow for each taxi to receive the same expected earnings

$$ q = Q/T $$

Then

$$ \pi_D^t = s.P^t.Q^t/T^t - (F_{D1} + F_{D2}.T^t) $$

Entry occurs to the point at which revenues and costs are equal ($\pi = 0$). Hence T* is determined by solution of a following quadratic.

$$ 0 = F_{D1}.T^t + F_{D2}.{T^t}^2 - s.P^t.Q^t $$

This has one positive solution at

$$ T^t = \frac{-F_{D1} + [(F_{D1})^2 + 4.F_{D2}.s.P^t.Q^t]^{1/2}}{2.F_{D2}} $$

For each shift, we reduce the number of taxis entering the shift from the number available until the shift constraint is met. That is:

$$ {\pi_D}^S = \sum_{All\ hours\ in\ shift\ t} {\pi_D}^t . \frac{T^t}{T^S} - C^S = 0 $$

N is determined as the point at which

$$\pi_O{}^y = \sum_{All\ shifts\ t} \pi_O{}^t . \frac{T^t}{N} - C^y = 0$$

Note that there is an additional constraint for the Melbourne taxi market in that $N \geq N^P$ where N^P is the number of perpetual licences. Where the equilibrium N would be lower than N^P then licence lease costs would be lower than in C^y and would become endogenous rather than determined as a Government imposed cost. This is allowed for in the modelling.

Costs associated with a time periods include labour costs to make it worthwhile for the driver to work, fuel costs, cleaning costs, risks of vehicle damage, operator cost of organising the driver and some part of insurance and vehicle costs. Costs also include GST payable.

Costs associated with annual taxi provision include the lease costs, some part of vehicle and insurance costs and network costs.

3. Quality (waiting time) equation

Waiting time for taxis is not constant across shifts (or within a shift). When there is very high demand, waiting time can increase substantially or people do not have any access to taxis, such as at New Year's Eve. Taxis can also be difficult to obtain on Friday and Saturday nights and peak times within a shift.

The academic literature has modelled waiting time based on search models of taxis finding passengers. This 'cruising market' waiting time model is derived from search equations and is as follows.

$$W = \frac{C_W}{V}$$

Where W is waiting time, C_W is a waiting time constant and V is the number of vacant taxis.

The number of vacant taxis can be rewritten as

$$V = T - Q.C_V$$

Where T is the total number of taxis working the shift, Q is total demand and C_V is a constant that relates kilometres of demand to hours of demand as a share of hours available for the shift.

The waiting time equation has the implication that if demand were twice as high for a shift and there were twice as many taxis operating, then waiting time would be lower.

The cruising market waiting time equation is likely to overstate waiting time in hours when there are few passengers and few taxis. In these periods (and for suburban locations), taxis are instead booked. This replaces the search costs (waiting) of taxis and passengers seeking each other with the financial costs associated with a dispatch system (and booking fee for passengers). Hence the 'waiting time' in these periods can be interpreted as the cost of lower accessibility. We would expect that this would still overstate accessibility costs, as the costs associated with dispatch/booking fees would be lower than the costs of waiting for booking to be preferred by customers.

Equilibrium

Time structures are: t is hour, s is shift and y is year.

$Q^t = C_Q{}^t - \beta_P{}^t . P^t - \beta_W{}^t . W^t$ (1)

$T^t = \frac{-F_{D1} + [(F_{D1})^2 + 4.F_{D2}.s.P^t.Q^t]^{1/2}}{2.F_{D2}}$ (2A) when $T^t < T^S$<=N or

$T^t = T^S$ otherwise (2B)

T^s is set based on: $\sum_{T^t \in T^t = T^s} \left[\frac{s.(P^t).Q^t}{T^s} - {F_{D1}}^t - {F_{D1}}^t.T^s\right] = C^s$ when $T^s < N$ (3a) or

$T^s = N$ (3b)

$W^t = \frac{C_W}{T^t - Q^t.C_V}$ (4)

N is from: $\sum_{T^t \in T^t = N} \left[\frac{(1-s).(P^t - c_O).Q^t}{N} - {F_O}^t\right] = C^y$ (5)

The endogenous variables are T^t, Q^t, W^t, T^s and N. This means that there are 3t+s+1 endogenous variables where t is the number of hours periods (12*14*3) and s is the number of shifts (14*3).

The number of equations is the same:

- t equations for each of (1) and (4)
- t equations for 2a and 2b
- s equations for 3a and 3b
- 1 equation for (5)

The number of equations is then 3*12*14*3+14*3+1 = 1555 equations.